Excitation of Low-Frequency Ion Acoustic Perturbations in the Presence of Stationary Lower Hybrid Turbulence

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Z. Naturforsch. **33a**, 121–123 (1978); received September 19, 1977

We have shown that stationary turbulence consisting of an ensemble of randomly phased small-amplitude lower hybrid waves becomes modulationally unstable against the ion acoustic perturbations along the external magnetic field. The growth rate of this instability is calculated.

Recently it has been shown that the lower hybrid turbulence consisting of an ensemble of randomly phased lower hybrid waves can either suppress the low-frequency Kelvin-Helmholtz instability [1] or become unstable with respect to adiabatic perturbations [2].

In this note, we show that unstable ion acoustic waves along the external magnetic field $B_0 \hat{z}$ occur owing to nonlinear interaction of lower hybrid turbulence with low-frequency ion acoustic perturbations. Specifically we present a general dispersion relation which governs the propagation of an ion acoustic perturbation in the presence of an ensemble of high-frequency lower hybrid wave packets.

The randomly distributed lower hybrid waves are assumed to propagate almost perpendicular to the external magnetic field and their dynamics is governed by the Liouville equation [3, 4]

$$\frac{\partial N_\mathrm{k}}{\partial t} + \boldsymbol{v}_\mathrm{g} \cdot \frac{\partial N_\mathrm{k}}{\partial \boldsymbol{r}} - \frac{\partial \omega_\mathrm{k}}{\partial \boldsymbol{r}} \cdot \frac{\partial N_\mathrm{k}}{\partial \boldsymbol{k}} = 0 \,, \qquad (1)$$

where $N_\mathrm{k} = \langle |E_\mathrm{k}|^2 \rangle$ is the effective distribution function of the quasi-particles, and $\boldsymbol{v}_\mathrm{g} = \partial \omega_\mathrm{k} / \partial \boldsymbol{k}$ is the group velocity. The frequency ω_k and the wave vector $\boldsymbol{k}$ satisfy the linear dispersion relation

$$\omega_\mathrm{k}^2 = \omega_\mathrm{LH}^2 \left(1 + \frac{k_\parallel^2}{k_\perp^2} \frac{M}{m} \right), \qquad (2)$$

where $\omega_\mathrm{LH}^2 = \omega_\mathrm{pi}^2 / (1 + \omega_\mathrm{pe}^2 / \omega_\mathrm{ce}^2)$, $\omega_{\mathrm{p}\alpha}$ and $\omega_{\mathrm{c}\alpha}$ are the plasma and gyro-frequencies respectively for the αth species, $\boldsymbol{k}_\parallel$ and $\boldsymbol{k}_\perp$ are the wave vector components along and across the external magnetic field $\boldsymbol{B}_0 (k_\parallel \equiv k_x, k_\perp \equiv k_z)$. The group velocity of the lower hybrid wave packet then becomes

$$\boldsymbol{v}_\mathrm{g} \cong \omega_\mathrm{LH} \frac{M}{m} \left[\frac{\boldsymbol{k}_\parallel}{k_\perp^2} - \frac{\boldsymbol{k}_\perp}{k_\perp^2} \left(\frac{k_\parallel}{k_\perp} \right)^2 \right]. \qquad (3)$$

Equations (1) and (2) reveal that a low-frequency $(\Omega, \boldsymbol{q})$ plasma motion can perturb N_k through the force term:

$$\frac{\partial \omega_\mathrm{k}}{\partial \boldsymbol{r}} = \frac{\omega_\mathrm{k}}{2 n_0} \frac{1}{1 + \omega_\mathrm{pe}^2 / \omega_\mathrm{ce}^2} \frac{\partial \tilde{n}_\mathrm{i}}{\partial \boldsymbol{r}} \,, \qquad (4)$$

where $\tilde{n}_\mathrm{i}$ is the low-frequency ion density perturbation associated with long wavelength ion acoustic waves. In Eq. (1) the space and time dependences are slow compared to space-time variations of lower hybrid turbulence. This implies that $\Omega \ll \omega_\mathrm{k}$ and $\boldsymbol{q} \ll \boldsymbol{k}$.

Letting

$$N_\mathrm{k} = N_\mathrm{k}^{(0)} + \tilde{N}_\mathrm{k} \exp(-i \Omega t + i \boldsymbol{q} \cdot \boldsymbol{r}) \,,$$
$$n_\mathrm{i} = n_0 + \tilde{n}_\mathrm{i} \exp(-i \Omega t + i \boldsymbol{q} \cdot \boldsymbol{r}) \,, \qquad (5)$$

we find from (1) and (4) the modified distribution

$$\tilde{N}_\mathrm{k} = - \frac{\tilde{n}_\mathrm{i}}{2 n_0} \frac{\omega_\mathrm{k}}{1 + \omega_\mathrm{pe}^2 / \omega_\mathrm{ce}^2} \frac{\boldsymbol{q} \cdot \partial N_\mathrm{k}^{(0)} / \partial \boldsymbol{k}}{\Omega - \boldsymbol{q} \cdot \boldsymbol{v}_\mathrm{g}} \,, \qquad (6)$$

where $N_\mathrm{k}^{(0)} = \langle |E_\mathrm{k}^{(0)}|^2 \rangle$ is the unperturbed distribution of the lower hybrid turbulence, and $\tilde{N}_\mathrm{k} \ll N_\mathrm{k}^{(0)}$, $\tilde{n}_\mathrm{i} \ll n_0$ have been assumed. The modified lower hybrid distribution then reacts back on the ion acoustic wave through the averaged pondermotive force along the z-direction

$$F_{\alpha z} = - \frac{\partial}{\partial z} \qquad (7)$$
$$\cdot \left[\left(\frac{\omega_{\mathrm{p}\alpha}}{\omega_\mathrm{k}} \right)^2 \frac{|\partial \varphi / \partial z|^2}{8 \pi} + \frac{\omega_{\mathrm{p}\alpha}^2}{\omega_\mathrm{k}^2 - \omega_{\mathrm{c}\alpha}^2} \frac{|\partial \varphi / \partial x|^2}{8 \pi} \right],$$

where φ is the high-frequency electric field potential. It should be noted that for real $\boldsymbol{k}$ the $\boldsymbol{E} \times \boldsymbol{B}_0$ force, due to the wave electric field in the y-direction, does

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not play any role in calculation of $F_{\alpha z}$. This low-frequency pondermotive force pushes the electrons and ions along the external magnetic field. The basic equations governing the propagation of low-frequency ion acoustic wave in the presence of lower hybrid turbulence are

$$\frac{\partial \tilde{n}_i}{\partial t} + n_0 \frac{\partial v_{iz}}{\partial z} = 0\,, \tag{8}$$

$$\frac{\partial v_{iz}}{\partial t} = -\frac{e}{M}\frac{\partial \varphi_s}{\partial z} + \frac{F_{iz}}{n_0 M}\,, \tag{9}$$

$$v_{Te}^2 \frac{\partial \tilde{n}_e}{\partial z} = \frac{e n_0}{m}\frac{\partial \varphi_s}{\partial z} + \frac{F_{ez}}{m}\,, \tag{10}$$

and

$$\tilde{n}_i = \tilde{n}_e\,, \tag{11}$$

where φ_s is the ambipolar potential. By eliminating $\varphi_s, \tilde{n}_e, v_{iz}$ from (8)—(11) and Fourier analyzing, we get

$$(\Omega^2 - q^2 c_s^2)\frac{\tilde{n}_i}{n_0} = \frac{q^2}{8\pi n_0 M} \cdot \left(\frac{\omega_{pe}^2}{\omega_k^2 - \omega_{ce}^2} + \frac{\omega_{pi}^2}{\omega_k^2}\right)\sum_{k'} \tilde{N}_{\tilde{k}}\,. \tag{12}$$

Here $c_s = (T_e/M)^{1/2}$ is the ion acoustic velocity. Substituting $\tilde{N}_k$ from (6) into (12), we find the following dispersion relation

$$\Omega^2 - q^2 c_s^2 = -\frac{q^2 L^2}{64\pi^3 n_0 M}\frac{\left(1 - \frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{\omega_{pe}^4}{\omega_{ce}^4}\right)\omega_{pi}}{(1+\omega_{pe}^2/\omega_{ce}^2)^{5/2}} \cdot \iint \mathrm{d}k_\parallel\, \mathrm{d}k_\perp \frac{\boldsymbol{q}\cdot \partial N_k^{(0)}/\partial \boldsymbol{k}}{\Omega - \boldsymbol{q}\cdot \boldsymbol{v}_g}\,. \tag{13}$$

In Eq. (13), while going from summation to an integration in $\boldsymbol{k}'$-space, we have used the relation

$$(2\pi/L)^2 \sum_{k'} \to \iint \mathrm{d}k_\parallel\, \mathrm{d}k_\perp\,,$$

where L is the size of the system.

For lower hybrid turbulence $k_\parallel/k_\perp \ll 1$ and for ion acoustic perturbations along z-direction $q_\perp = 0$. In such a case we can rewrite (9) using (3) as

$$\Omega^2 - q_\parallel^2 c_s^2 = -\beta \iint \mathrm{d}k_\parallel\, \mathrm{d}k_\perp \frac{q_\parallel \cdot \partial N_k^{(0)}/\partial k_\parallel}{\Omega - q_\parallel \alpha}\,, \tag{14}$$

where

$$\alpha = \frac{k_\parallel}{k_\perp}\frac{M}{m}\frac{\omega_{LH}}{k_\perp}$$

and

$$\beta = \frac{\omega_{pi} q_\parallel^2 L^2}{64\pi^3 n_0 M}\frac{1 - \omega_{pe}^2/\omega_{ce}^2 - \omega_{pe}^4/\omega_{ce}^4}{(1+\omega_{pe}^2/\omega_{ce}^2)^{5/2}}\,. \tag{15}$$

We take the spectrum of weakly interacting lower hybrid quasi-particles in the Gaussian form [2], i.e.

$$N_k^{(0)} = \frac{2\pi W}{L^2 \Delta_\parallel \Delta_\perp}\exp\left[-\frac{(k_\parallel - k_{\parallel 0})^2}{2\Delta_\parallel^2}\right] \cdot \exp\left[-\frac{(k_\perp - k_{\perp 0})^2}{2\Delta_\perp^2}\right], \tag{16}$$

where $\Delta_\parallel$ and $\Delta_\perp$ are the thermal spreads of the distribution, $k_{\parallel 0}$ and $k_{\perp 0}$ are the components of the mean wave vector of the spectrum, and W is the total energy density.

Inserting (16) into (14), we obtain

$$\Omega^2 - q_\parallel^2 c_s^2 = \frac{q_\parallel^2 \omega_{pi} W}{32\pi^2 n_0 M \Delta_\parallel \Delta_\perp} \frac{1 - \omega_{pe}^2/\omega_{ce}^2 - \omega_{pe}^4/\omega_{ce}^4}{(1+\omega_{pe}^2/\omega_{ce}^2)^{5/2}} \cdot \iint \mathrm{d}k_\parallel\, \mathrm{d}k_\perp \frac{q_\parallel (k_\parallel - k_{\parallel 0})/\Delta_\parallel^2}{\Omega - \frac{k_\parallel q_\parallel}{k_\perp^2}\frac{M}{m}\omega_{LH}} \cdot \exp\left[-\frac{(k_\parallel - k_{\parallel 0})^2}{2\Delta_\parallel^2}\right]\exp\left[-\frac{(k_\perp - k_{\perp 0})^2}{2\Delta_\perp^2}\right]. \tag{17}$$

After performing double integration over $\boldsymbol{k}$-space, we get from (17) a dispersion relation

$$\Omega^2 - q_\parallel^2 c_s^2 = \frac{q_\parallel^2 c_s^2 W}{16\pi n_0 T_e}\frac{m}{M}\left(\frac{\Delta_\perp}{\Delta_\parallel}\right)^2 \cdot \frac{1 - \omega_{pe}^2/\omega_{ce}^2 - \omega_{pe}^4/\omega_{ce}^4}{(1+\omega_{pe}^2/\omega_{ce}^2)^2}\left\{\left[1 - \mathrm{i}\left(\frac{\pi}{2}\right)^{1/2}\frac{k_{\parallel 0}}{\Delta_\parallel}\right] \cdot \left[1 + \left(\frac{k_{\perp 0}}{\Delta_\perp}\right)^2\right] + i\left(\frac{\pi}{2}\right)^{1/2}\frac{m}{M}\frac{\Omega}{\omega_{LH}}\frac{\Delta_\perp^2}{q_\parallel \Delta_\parallel} \cdot \left[3 + 6\left(\frac{k_{\perp 0}}{\Delta_\perp}\right)^2 + \left(\frac{k_{\perp 0}}{\Delta_\perp}\right)^4\right]\right\}. \tag{18}$$

By substituting $\Omega = q_\parallel c_s + i\gamma$, $\gamma \ll q_\parallel c_s$, we obtain from (18)

$$\gamma = \frac{q_\parallel c_s W}{32\pi n_0 T_e}\frac{m}{M}\left(\frac{\Delta_\perp}{\Delta_\parallel}\right)^2\left(\frac{\pi}{2}\right)^{1/2}\frac{k_{\parallel 0}}{\Delta_\parallel} \cdot \frac{1 - \omega_{pe}^2/\omega_{ce}^2 - \omega_{pe}^4/\omega_{ce}^4}{(1+\omega_{pe}^2/\omega_{ce}^2)^2}\cdot\left\{-\left[1 + \left(\frac{k_{\perp 0}}{\Delta_\perp}\right)^2\right] + \frac{m}{M} \cdot \frac{k_{\parallel 0} c_s}{\omega_{LH}}\left(\frac{k_{\perp 0}}{k_{\parallel 0}}\right)^2\left(\frac{\Delta_\perp}{k_{\perp 0}}\right)^2\left[3 + 6\left(\frac{k_{\perp 0}}{\Delta_\perp}\right)^2 + \left(\frac{k_{\perp 0}}{\Delta_\perp}\right)^4\right]\right\}, \tag{19}$$

where we have neglected the small frequency shift caused by the stationary turbulence.

For $\omega_{\mathrm{pe}}/\omega_{\mathrm{ce}} \approx 1$, $\gamma > 0$ if

$$1 + \left(\frac{k_{\perp 0}}{\Delta_\perp}\right)^2 > \frac{m}{M} \frac{k_{\parallel 0}\, c_{\mathrm{s}}}{\omega_{\mathrm{LH}}} \cdot \left(\frac{k_{\perp 0}}{k_{\parallel 0}}\right)^2 \left(\frac{\Delta_\perp}{k_{\perp 0}}\right)^2 \cdot \left[3 + 6\left(\frac{k_{\perp 0}}{\Delta_\perp}\right)^2 + \left(\frac{k_{\perp 0}}{\Delta_\perp}\right)^4\right]$$

and the excitation of ion acoustic waves along the external magnetic field by the lower hybrid turbulence is possible for any value of W.

In conclusion, we have shown that the lower hybrid turbulence may become unstable with respect to low-frequency long wavelength ion acoustic perturbations along the external magnetic field.

Acknowledgement

The authors acknowledge the Alexander von Humboldt Foundation for the award of Senior Fellowships.

[1] M. Sinha and A. K. Sundaram, Plasma Phys. **19**, 407 (1977).
[2] S. G. Tagare and I. Zhelyazkov, Phys. Lett. **64** A, 56 (1977).
[3] A. A. Vedenov and L. I. Rudakov, Sov. Phys.-Doklady **9**, 1073 (1965).
[4] B. B. Kadomtsev, Plasma Turbulence, Academic Press, New York 1965.